Enrollment No: ______________________ Exam Seat No:______________________

C.U.SHAH UNIVERSITY

SummerExamination-2019

Subject Name: Mathematics - II

Subject Code: 4SC02MAT1 **Branch: B.Sc. (All)**

Semester: 2 **Date: 29/04/2019** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) Find polar form of $(1 + i)$ 1

b) If $y = \cos\theta + i\, sin\theta$ then find value of $y + \frac{1}{y}$. 1

c) If z is purely imaginary then $z \neq \bar{z}$. True/False. 1

d) The number i^i is purely imaginary number. True/False. 1

e) Define: Reciprocal cone. 1

f) Write standard form of Ellipsoid. 1

g) Solve: $(D-1)^2 y = 0$ 2

h) Find $\frac{1}{D-a}\, k$, where k is constant. 2

i) Write tangency condition for cone. 2

j) Prove that $\sin ix = i \sin hx$. 2

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Consider the equation $(D^n + a_1 D^{n-1} + a_2 D^{n-2} + \cdots + a_n)y = 0$ where $a_i^{'}s$ are constant. If $m_1, m_2, \ldots m_n$ are real and different roots of A.E. Then prove that $y = c_1 e^{m_1 x} + c_2 e^{m_2 x} + \cdots . + c_n e^{m_n x}$ is complete solution of given equation. **6**

b) Find Particular Integral of $(D-1)(D-2)y = e^{-2x} + e^x + \sin 2x + \cos 3x$. **4**

c) Solve: $(D^4 - 1)y = e^x \cos x$. **4**

Q-3 **Attempt all questions** **(14)**

a) Solve: $x^2 \frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 20y = (x+1)^2$. **5**

b) Solve: $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 2 \sin \log(1+x)$. **5**

c) Solve: $\frac{dx}{dt} + y = e^t, \frac{dy}{dt} + x = e^{-t}$. **4**

Q-4 **Attempt all questions** **(14)**

a) Prove that $(\cos\theta + i \sin\theta)^n = \cos n\theta + i \sin n\theta \;\forall\, n \in Q$. **5**

b) Solve: $(D^2 - 1)y = \cos hx \cos x$. **5**

	c)	Prove that $\frac{1}{D-m}X = e^{mx}\int e^{-mx}\, X\, dx$.	4
Q-5		**Attempt all questions**	**(14)**
	a)	Prove that $\left(\frac{1+sin\theta\ +icos\theta}{1+sin\theta\ -icos\theta}\right)^n = \cos\left(\frac{n\pi}{2} - n\theta\right) + i\sin\left(\frac{n\pi}{2} - n\theta\right)$.	5
	b)	Solve: $x^4 - x^3 + x^2 - x + 1 = 0$.	5
	c)	Prove that $(a+ib)^{\frac{m}{n}} + (a-ib)^{\frac{m}{n}} = 2\,(a^2+b^2)^{\frac{m}{2n}}\cos(\frac{m}{n}\tan^{-1}\frac{b}{a})$.	4
Q-6		**Attempt all questions**	**(14)**
	a)	Expand $\sin^5\theta$ in a series of sine multiple of θ.	5
	b)	If $\cos^{-1}(u+iv) = x+iy$ then prove that (1) $u^2\sec^2 x - v^2 cosec^2 x = 1$. (2) $u^2 \sec h^2 y + v^2\,\text{cosech}^2\, y = 1$.	5
	c)	Find real and imaginary part of $(i)^i$.	4
Q-7		**Attempt all questions**	**(14)**
	a)	Prove that equation of cone which passes through (α,β,γ) and having guiding curve conic is $a(\alpha z - x\gamma)^2 + b(\beta z - y\gamma)^2 + c(z-\gamma)^2 + 2h(\alpha z - xy)$ $(\beta z - y\gamma) + 2g(\alpha z - x\gamma)(z-\gamma) + 2f(\beta z - y\gamma)(z-\gamma) = 0$.	6
	b)	Find equation of cone whose vertex is $(-1,-2,-3)$ and base curve is $x^2 + z^2 = 1, y = 0$.	5
	c)	Identify the surface $x^2 + y^2 + z^2 + 4x - 6y = 3$.	3
Q-8		**Attempt all questions**	**(14)**
	a)	Prove that equation of right circular cylinder having axis line $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ and radius r is $(x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2 - \frac{[l(x-\alpha)+m(y-\beta)+n(z-\gamma)]^2}{l^2+m^2+n^2}$ $= r^2$.	6
	b)	Find equation of cylinder whose generators are parallel to $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and having guiding curve $x^2 + y^2 = 16, z = 0$.	5
	c)	Find reciprocal cone of $ax^2 + by^2 + cz^2 = 0$.	3